WORKFORCE DEVELOPMENT AUTHORITY
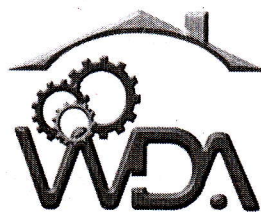
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ADVANCED LEVEL NATIONAL EXAMINATIONS, 2017, TECHNICAL AND PROFESSIONAL STUDIES

EXAM TITLE: MATHEMATICS A

OPTIONS: Computer Science (CSC)

Computer Science and Management (CSM)

DURATION: 3 hours

INSTRUCTIONS:

The paper is composed of **the following Sections**:

Section I: Sixteen (16) compulsory questions.	**55 marks**	
Section II: Attempt **any three (3)** out of five questions.	**45 marks**	

Note:

Every candidate is required to carefully comply with the above instructions. Penalty measures will be applied on their strict consideration.

Section I. Sixteen (16) Compulsory questions   55 marks

01. If $\vec{a} = 7\,\vec{i} + 8\vec{j}$ and $\vec{c} = 5\vec{i} - 2\vec{j}$

Find the scalar product $\vec{a} \cdot \vec{c}$ **2 marks**

02. Solve $2log_b(x) = log_b(4) + log_b(x - 1)$.   **3 marks**

03. Find the distance between parallel planes:

$10X - 12Y + 5Z - 8 = 0$ and $10X - 12Y = 5Z + 4$

$Q(1, 1, 2)$ is a point in the first plane.   **2 marks**

04. Find the complex number z such that $5iz + \bar{z} + 16 = 8i$

Give your answer in the form a + bi, where a and b are real numbers.

5 marks

05. Find the equation of the plane containing the three points

(1,2,3),(2,0,4) and (3,3,1), (2,0,4) and (3,3,1)   **3 marks**

06. Use de Moivre's theorem to show that

$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$ **5 marks**

07. Find $\int \sin^5 x \cos^2 x \sin x \, dx$ **5 marks**

08. Compute the truth table for (p∨q) ⟶ (p∧q)   **5 marks**

09. Find the general solution of the differential equation

$y'' - 11y' + 30y = 0$ **1 mark**

10. Show that the function $y = (x + 1) - \frac{1}{3}e^x$ is a solution to the first order

initial value problem $\frac{dy}{dx} = y - x$; $y(0) = \frac{2}{3}$ **3 marks**

11. The first term of a geometric series is 54 and the common ratio of the

series is $\frac{8}{9}$

i) Find the sum to infinity of the series
ii) Find the second term of the series

iii) Show that the series can be written in the form $\frac{2^a}{3^b}$, where a and b are

integers.

4 marks

12. If the matrix A= $\begin{pmatrix} 2 & 7 \\ 1 & 4 \end{pmatrix}$

    a) Calculate the matrix B which is an inverse of A.

    b) The product of A and B.   **4 marks**

13. Calculate $\int x^2 e^{x^3} dx$ **3 marks**

14. Given $f(x) = x - 5$; $g(x) = x^2$ and $k(x) = \sqrt{x + 2}$

    a. Find $(gof)(x)$ and write its domain in interval notation.

    b. Find $(kof)(x)$ and write its domain in interval notation.   **4 marks**

15. Solve for x the equation $(2^x)^x + 3x = \frac{1}{4}$ **3 marks**

16. Given $\vec{V} = 3\vec{i} + 5\vec{j}$,

    find:

        i) $\|v\|$

        ii) θ where $0 \le \theta \le 360^o$

           Round off θ to the nearest tenth of the degree.   **3 marks**

Section II. Choose and answer any three (3) questions   **45 marks**

17. Given the function $f(x) = \frac{x-4}{x+3}$

    a. Determine the domain of definition;

    b. Find the intercept point with axes;

    c. Periodicity;

    d. Find the asymptotes to the curve;

    e. Compute the first derivative and study its sign;

    f. Compute the second derivative and study its sign;

    g. Sketch the graph of f(x)

                                                      15 marks

18. Solve the following series of questions   **15 marks**

    a. Consider the plane with the direction vectors $\vec{a} = [8,-5, 4]$ and $\vec{b} = [1,-3,-2]$ through Po (3, 7, 0).

        i) Write the vector and parametric equation of the plane.

        ii) Determine if the point Q (-10, 8,-6) is on the plane.

    b. Consider the plane that has normal vector

        $\vec{n} = (3,-2, 5)$ and contains the points Po (1, 2,-3)

        i) Write the scalar equation of the plane.

        ii) Is the vector $\vec{a} = (4, 1,-2)$ parallel to the plane?

    c. A circle with centre C has an equation $x^2 + y^2 - 10x + 12y + 41 = 0$

        The point B (3,-2) lies on the circle

        (1) Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k$$

        (2) Write down the coordinates of C

19. An ellipse E has equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$

    a) Sketch the ellipse E, showing the values of the intercepts on the coordinate axes.

    b) Given that the line with equation $y = x + t$ intersects the ellipse E at two distinct points, show that $-5 < t < 5$.

    c) The ellipse E is translated by the vector $\begin{vmatrix} a \\ b \end{vmatrix}$ to form another ellipse whose equation is $9x^2 + 16y^2 + 18x - 64y = d$. Find the values of the constants a, b and d.

    d) Hence find an equation for each of the two tangents to the ellipse $9x^2 + 1616y^2 + 18x - 64y = d$ that are parallel to the line $y = x$

                                                          15 marks

20. Find solutions to the following series of questions   **15 marks**

    a. An arithmetic series the first term a and common difference d the sum of the first five terms of the series is 575

    (1) Show that $a + 2d = 115$

    (2) Given also that the 10th term of the series is 87, find the value of d.

    (3) The nth term of the series is u_n

        Given that $u_k > 0$ and $u_k + 1 < 0$,   Find $\sum_{n=1}^{k} u_n$

    b. Calculate the $\lim\limits_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x-2}$

21. Find solutions to the following series of questions   **15 marks**

    a. Consider the plane with direction vectors $\vec{a} = [8, -5, 4]$ and $\vec{b} = [1, -3, -2]$ through $P_0(3,7,0)$

        i) Write the vector equation of the plane.

        ii) Write the parametric equation of the plane.

        iii) Determine if the point M $(-10, 8, -6)$ is on the plane.

        iv) Find x-intercept of the plane.

    b. Use Cramer's Rule to solve the following system:

$$2x + 5y - z = 4$$
$$x - 3y + 2z = 3$$
$$3x - 2y + z = 8$$